### **Expressing Equilibrium Constants for Chemical Equations**

Express the equilibrium constant for the chemical equation:

 $CH_3OH(g) \Longrightarrow CO(g) + 2 H_2(g)$ 

### **Solution**

The equilibrium constant is the equilibrium concentrations of the products raised to their stoichiometric coefficients divided by the equilibrium concentrations of the reactants raised to their stoichiometric coefficients.

 $K = \frac{[\text{CO}][\text{H}_2]^2}{[\text{CH}_3\text{OH}]}$ 

### **For Practice**

Express the equilibrium constant for the combustion of propane:

 $C_3H_8(g) + 5 O_2(g) \Longrightarrow 3 CO_2(g) + 4 H_2O(g)$ 

### Manipulating the Equilibrium Constant to Reflect Changes in the Chemical Equation

Consider the chemical equation and equilibrium constant for the synthesis of ammonia at 25 °C:

 $N_2(g) + 3 H_2(g) \implies 2 NH_3(g) \qquad K = 5.6 \times 10^5$ 

Calculate the equilibrium constant for the following reaction at 25 °C :

$$\mathrm{NH}_3(g) \Longrightarrow \tfrac{1}{2}\mathrm{N}_2(g) + \tfrac{3}{2}\mathrm{H}_2(g) \qquad K' = ?$$

#### **Solution**

You want to manipulate the given reaction and value of K to obtain the desired reaction and value of K. You can see that the given reaction is the reverse of the desired reaction, and its coefficients are twice those of the desired reaction.

Begin by reversing the given reaction and taking the inverse of the value of K.

$$N_{2}(g) + 3 H_{2}(g) \rightleftharpoons 2 NH_{3}(g) \qquad K = 5.6 \times 10^{5}$$
  

$$2 NH_{3}(g) \rightleftharpoons N_{2}(g) + 3 H_{2}(g) \qquad K_{\text{reverse}} = \frac{1}{5.6 \times 10^{5}}$$

Next, multiply the reaction by  $\frac{1}{2}$  and raise the equilibrium constant to the  $\frac{1}{2}$  power.

$$\mathrm{NH}_3(g) \Longrightarrow \frac{1}{2} \mathrm{N}_2(g) + \frac{3}{2} \mathrm{H}_2(g)$$
$$K' = K_{\mathrm{reverse}}^{1/2} = \left(\frac{1}{5.6 \times 10^5}\right)^{1/2}$$

Calculate the value of *K*'.

 $K' = 1.3 \times 10^{-3}$ 

### Manipulating the Equilibrium Constant to Reflect Changes in the Chemical Equation

Continued

#### **For Practice**

Consider the following chemical equation and equilibrium constant at 25 °C:

 $2 \operatorname{COF}_2(g) \Longrightarrow \operatorname{CO}_2(g) + \operatorname{CF}_4(g) \quad K = 2.2 \times 10^6$ 

Calculate the equilibrium constant for the following reaction at 25 °C:

$$2 \operatorname{CO}_2(g) + 2 \operatorname{CF}_4(g) \Longrightarrow 4 \operatorname{COF}_2(g)$$
  $K' = ?$ 

Predict the equilibrium constant for the first reaction shown here given the equilibrium constants for the second and third reactions:

$$\begin{array}{ll} \text{CO}_2(g) + 3 \text{ H}_2(g) & \Longrightarrow & \text{CH}_3\text{OH}(g) + \text{H}_2\text{O}(g) \\ \text{CO}(g) + \text{H}_2\text{O}(g) & \Longrightarrow & \text{CO}_2(g) + \text{H}_2(g) \\ \text{CO}(g) + 2 \text{ H}_2(g) & \rightleftharpoons & \text{CH}_3\text{OH}(g) \end{array} \qquad \begin{array}{ll} K_1 = ? \\ K_2 = 1.0 \times 10^5 \\ K_3 = 1.4 \times 10^7 \end{array}$$

### Relating $K_{\rm p}$ and $K_{\rm c}$

Nitrogen monoxide, a pollutant in automobile exhaust, is oxidized to nitrogen dioxide in the atmosphere according to the equation:

 $2 \operatorname{NO}(g) + \operatorname{O}_2(g) \implies 2 \operatorname{NO}_2(g) \qquad K_p = 2.2 \times 10^{12} \text{ at } 25 \text{ °C}$ 

Find  $K_c$  for this reaction.

### Sort

You are given  $K_p$  for the reaction and asked to find  $K_c$ . **Given:**  $K_p = 2.2 \times 10^{12}$ **Find:**  $K_c$ 

### Strategize

Use Equation 16.2 to relate  $K_p$  and  $K_c$ .

### Equation

 $K_{\rm p} = K_{\rm c}({\rm RT})^{\Delta n}$ 

#### Solve

Solve the equation for  $K_c$ .

Calculate  $\Delta n$ .

Substitute the required quantities to calculate  $K_c$ . The temperature must be in kelvins. The units are dropped when reporting  $K_c$ , as described later in this section.

### Relating $K_{\rm p}$ and $K_{\rm c}$

Continued

### **Solution**

$$K_{\rm c} = \frac{K_{\rm p}}{(RT)^{\Delta n}}$$
$$\Delta n = 2 - 3 = -1$$
$$K_{\rm c} = \frac{2.2 \times 10^{12}}{\left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \times 298 \text{ K}\right)^{-1}}$$
$$= 5.4 \times 10^{13}$$

#### Check

The easiest way to check this answer is to substitute it back and confirm that you get the original value for  $K_p$ .

$$K_{\rm p} = K_{\rm c}(RT)^{\Delta n}$$
$$= 5.4 \times 10^{13} \left( 0.08206 \,\frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \times 298 \,\text{K} \right)^{-1}$$

### **For Practice**

$$= 2.2 \times 10^{12}$$

Consider the following reaction and corresponding value of  $K_c$ :

$$H_2(g) + I_2(g) \qquad \qquad 2 \text{ HI}(g) \qquad \qquad K_c = 6.2 \times 10^2 \text{ at } 25 \text{ °C}$$

What is the value of  $K_p$  at this temperature?

# Writing Equilibrium Expressions for Reactions Involving a Solid or a Liquid

Write an expression for the equilibrium constant  $(K_c)$  for this chemical equation:

 $CaCO_3(s) \Longrightarrow CaO(s) + CO_2(g)$ 

#### **Solution**

Since  $CaCO_3(s)$  and CaO(s) are both solids, omit them from the equilibrium expression.

 $K_{\rm c} = [\rm CO_2]$ 

#### **For More Practice**

Write an equilibrium expression  $(K_c)$  for the equation:

 $4 \operatorname{HCl}(g) + \operatorname{O}_2(g) \Longrightarrow 2 \operatorname{H}_2\operatorname{O}(\ell) + 2 \operatorname{Cl}_2(g)$ 

Consider the following reaction:

 $CO(g) + 2 H_2(g) \Longrightarrow CH_3OH(g)$ 

A reaction mixture at 780 °C initially contains [CO] = 0.500 M and  $[H_2] = 1.00$  M. At equilibrium, the CO concentration is found to be 0.15 M. What is the value of the equilibrium constant?

#### **How To**

#### Find Equilibrium Constants from Experimental Concentration Measurements

To solve these types of problems, follow the given procedure.

**Step 1 Using the balanced equation as a guide, prepare an ICE table showing the known initial concentrations and equilibrium concentrations of the reactants and products.** Leave space in the middle of the table for determining the changes in concentration that occur during the reaction.

	[CO]	[H <sub>2</sub> ]	[CH <sub>3</sub> OH]
Initial	0.500	1.00	0.00
<b>C</b> hange			
Equil	0.15		

$CO(g) + 2 H_2(g) \Longrightarrow CH_3OH(g)$	$(g) + 2 H_2(g) \equiv$	$\Rightarrow$ CH <sub>3</sub> OH(g)
--	-------------------------	-------------------------------------

Continued

**Step 2** For the reactant or product whose concentration is known both initially and at equilibrium, calculate the change in concentration that occurs.

			5 (0)
	[CO]	[H <sub>2</sub> ]	[CH <sub>3</sub> OH]
Initial	0.500	1.00	0.00
<b>C</b> hange	-0.35		
Equil	0.15		

$$CO(g) + 2 H_2(g) \Longrightarrow CH_3OH(g)$$

**Step 3** Use the change you calculated in step 2 and the stoichiometric relationships from the balanced chemical equation to determine the changes in concentration of all other reactants and products. Since reactants are consumed during the reaction, the changes in their concentrations are negative. Since products are formed, the changes in their concentrations are positive .

= -2(3) = $-2(3)$ = $-3$ = $-(3)$			
	[CO]	[H <sub>2</sub> ]	[CH <sub>3</sub> OH]
Initial	0.500	1.00	0.00
<b>C</b> hange	-0.35	- <b>2</b> (0.35)	+0.35
Equil	0.15		

 $CO(g) + 2 H_2(g) \Longrightarrow CH_3OH(g)$ 

Continued

**Step 4** Sum each column for each reactant and product to determine the equilibrium concentrations.

	[CO]	[H <sub>2</sub> ]	[CH <sub>3</sub> OH]
Initial	0.500	1.00	0.00
<b>C</b> hange	-0.35	-0.70	+0.35
Equil	0.15	0.30	0.35

**Step 5** Use the balanced equation to write an expression for the equilibrium constant and substitute the equilibrium concentrations to calculate *K*.

$$K_{\rm c} = \frac{[\rm CH_3OH]}{[\rm CO][\rm H_2]^2} \\ = \frac{0.35}{(0.15)(0.30)^2} \\ = 26$$

Continued

### **For Practice**

The reaction between CO and  $H_2$  is carried out at a different temperature with initial concentrations of [CO] = 0.27 M and  $[H_2] = 0.49$  M. At equilibrium, the concentration of CH<sub>3</sub>OH is 0.11 M. Find the equilibrium constant at this temperature

Consider the following reaction:

 $2 \operatorname{CH}_4(g) \Longrightarrow \operatorname{C}_2\operatorname{H}_2(g) + 3 \operatorname{H}_2(g)$ 

A reaction mixture at 1700 °C initially contains  $[CH_4] = 0.115$  M. At equilibrium, the mixture contains  $[C_2H_2] = 0.035$  M. What is the value of the equilibrium constant?

### **How To**

#### Find Equilibrium Constants from Experimental Concentration Measurements

To solve these types of problems, follow the given procedure.

**Step 1 Using the balanced equation as a guide, prepare an ICE table showing the known initial concentrations and equilibrium concentrations of the reactants and products.** Leave space in the middle of the table for determining the changes in concentration that occur during the reaction.

	[CH <sub>4</sub> ]	$[C_2H_2]$	[H <sub>2</sub> ]
Initial	0.115	0.00	0.00
Change			
Equil		0.035	

Continued

For the reactant or product whose concentration is known both initially and at equilibrium, calculate Step 2 the change in concentration that occurs.

$2 \operatorname{CH}_4(\mathfrak{z}) \longleftarrow \operatorname{C}_2\operatorname{H}_2(\mathfrak{z}) + \operatorname{SH}_2(\mathfrak{z})$			
	[CH <sub>4</sub> ]	$[C_2H_2]$	[H <sub>2</sub> ]
Initial	0.115	0.00	0.00
<b>C</b> hange		+0.035	
Equil		0.035	

 $2 CH_1(\sigma) \longrightarrow C_2H_2(\sigma) + 3 H_2(\sigma)$ 

Use the change you calculated in step 2 and the stoichiometric relationships from the balanced chemical Step 3 equation to determine the changes in concentration of all other reactants and products. Since reactants are consumed during the reaction, the changes in their concentrations are negative. Since products are formed, the changes in their concentrations are positive.

$$2 \operatorname{CH}_4(g) \Longrightarrow \operatorname{C}_2\operatorname{H}_2(g) + 3 \operatorname{H}_2(g)$$

	[CH₄]	$[C_2H_2]$	[H <sub>2</sub> ]
Initial	0.115	0.00	0.00
<b>C</b> hange	- <b>2</b> (0.035)	+0.035	+3(0.035)
Equil		0.035	

Continued

**Step 4** Sum each column for each reactant and product to determine the equilibrium concentrations.

	[CH <sub>4</sub> ]	$[C_2H_2]$	[H <sub>2</sub> ]
Initial	0.115	0.00	0.00
<b>C</b> hange	-0.070	+0.035	+0.105
Equil	0.045	0.035	0.105

**Step 5** Use the balanced equation to write an expression for the equilibrium constant and substitute the equilibrium concentrations to calculate *K*.

$$K_{\rm c} = \frac{[{\rm C}_2 {\rm H}_2] [{\rm H}_2]^3}{[{\rm C} {\rm H}_4]^2}$$
$$= \frac{(0.035)(0.105)^3}{(0.045)^2}$$
$$= 0.020$$

Continued

#### **For Practice**

The reaction of  $CH_4$  is carried out at a different temperature with an initial concentration of  $[CH_4] = 0.087$  M. At equilibrium, the concentration of  $H_2$  is 0.012 M. Find the equilibrium constant at this temperature.

### Predicting the Direction of a Reaction by Comparing Q and K

Consider the reaction and its equilibrium constant at 25.0 °C:

 $I_2(g) + Cl_2(g) \Longrightarrow 2 ICl(g) \qquad K_p = 81.9$ 

A reaction mixture contains  $P_{I_2} = 0.114$  atm,  $P_{CI_2} = 0.102$  atm, and  $P_{ICI} = 0.355$  atm. Is the reaction mixture at equilibrium? If not, in which direction will the reaction proceed?

### **Solution**

To determine the progress of the reaction relative to the equilibrium state, first calculate Q.

$$Q_{\rm p} = \left(\frac{P_{\rm ICl}^2}{P_{\rm I_2}P_{\rm Cl_2}}\right)$$
$$= \frac{(0.355)^2}{(0.114)(0.102)}$$
$$= 10.8$$

Compare Q to K.

$$Q_{\rm p} = 10.8; K_{\rm p} = 81.9$$

Since  $Q_p < K_p$ , the reaction is not at equilibrium and will proceed to the right.

### Predicting the Direction of a Reaction by Comparing Q and K

Continued

#### **For Practice**

Consider the reaction and its equilibrium constant:

 $N_2O_4(g) \implies 2 NO_2(g) K_c = 5.85 \times 10^{-3}$  (at some temperature)

A reaction mixture contains  $[NO_2] = 0.0255$  M and  $[N_2O_4] = 0.0331$  M. Calculate  $Q_c$  and determine the direction in which the reaction will proceed.

# Finding Equilibrium Concentrations When You Know the Equilibrium Constant and All but One of the Equilibrium Concentrations of the Reactants and Products

Consider the following reaction:

 $2 \operatorname{COF}_2(g) \Longrightarrow \operatorname{CO}_2(g) + \operatorname{CF}_4(g) \qquad K_c = 2.00 \text{ at } 1000 \ ^\circ \text{C}$ 

In an equilibrium mixture, the concentration of  $COF_2$  is 0.255 M and the concentration of  $CF_4$  is 0.118 M. What is the equilibrium concentration of  $CO_2$ ?

### Sort

You are given the equilibrium constant of a chemical reaction, together with the equilibrium concentrations of the reactant and one product. You are asked to find the equilibrium concentration of the other product.

Given:  $[COF_2] = 0.255 \text{ M}$  $[CF_4] = 0.118 \text{ M}$  $K_c = 2.00$ Find:  $[CO_2]$ 

### **Strategize**

Calculate the concentration of the product using the given quantities and the expression for  $K_c$ .

# Finding Equilibrium Concentrations When You Know the Equilibrium Constant and All but One of the Equilibrium Concentrations of the Reactants and Products

Continued

#### **Conceptual Plan**



#### Solve

Solve the equilibrium expression for  $[CO_2]$  and then substitute in the appropriate values to calculate it.

**Solution** 

$$[CO_2] = K_c \frac{[COF_2]^2}{[CF_4]}$$
$$[CO_2] = 2.00 \left(\frac{(0.255)^2}{0.118}\right) = 1.10 \text{ M}$$

# Finding Equilibrium Concentrations When You Know the Equilibrium Constant and All but One of the Equilibrium Concentrations of the Reactants and Products

Continued

### Check

Check your answer by mentally substituting the given values of  $[COF_2]$  and  $[CF_4]$  as well as your calculated value for  $CO_2$  back into the equilibrium expression:

$$K_{\rm c} = \frac{[\rm CO_2][\rm CF_4]}{[\rm COF_2]^2}$$

[CO<sub>2</sub>] is roughly equal to 1. [COF<sub>2</sub>]<sup>2</sup>  $\approx$  0.06 and [CF<sub>4</sub>]  $\approx$  0.12. Therefore,  $K_c$  is approximately 2, as given in the problem.

### For Practice 16.8

Diatomic iodine [I<sub>2</sub>] decomposes at high temperature to form I atoms according to the reaction:

$$I_2(g)$$
 2 I(g)  $K_c = 0.011$  at 1200 °C

In an equilibrium mixture, the concentration of  $I_2$  is 0.10 M. What is the equilibrium concentration of I?

Consider the reaction:

 $N_2(g) + O_2(g) \Longrightarrow 2 NO(g)$ 

 $K_{\rm c} = 0.10 \; ({\rm at} \; 2000 \; {}^{\circ}{\rm C})$ 

A reaction mixture at 2000 °C initially contains  $[N_2] = 0.200$  M and  $[O_2] = 0.200$  M. Find the equilibrium concentrations of the reactants and product at this temperature.

### How To

#### Find Equilibrium Concentrations from Initial Concentrations and the Equilibrium Constant

To solve these types of problems, follow the given procedure.

**Step 1 Using the balanced equation as a guide, prepare a table showing the known initial concentrations of the reactants and products.** Leave room in the table for the changes in concentrations and for the equilibrium concentrations.

$$N_2(g) + O_2(g) \Longrightarrow 2 \operatorname{NO}(g)$$

	[N <sub>2</sub> ]	[O <sub>2</sub> ]	[NO]
Initial	0.200	0.200	0.00
<b>C</b> hange			
Equil			

Continued

**Step 2** Use the initial concentrations to calculate the reaction quotient (*Q*) for the initial concentrations. Compare *Q* to *K* to predict the direction in which the reaction proceeds.

$$Q_{c} = \frac{[NO]^{2}}{[N_{2}][O_{2}]} = \frac{(0.00)^{2}}{(0.200)(0.200)}$$
$$= 0$$

Q < K; therefore, the reaction will proceed to the right.

**Step 3 Represent the change in the concentration of one of the reactants or products with the variable** *x***. Define the changes in the concentrations of the other reactants or products in terms of** *x***.** It is usually most convenient to let *x* represent the change in concentration of the reactant or product with the smallest stoichiometric coefficient.

$$N_2(g) + O_2(g) \Longrightarrow 2 NO(g)$$

	[N <sub>2</sub> ]	[O <sub>2</sub> ]	[NO]
Initial	0.200	0.200	0.00
Change	—x	—x	+2x
Equil			

Continued

**Step 4** Sum each column for each reactant and each product to determine the equilibrium concentrations in terms of the initial concentrations and the variable *x*.

$$N_2(g) + O_2(g) \Longrightarrow 2 NO(g)$$

	[N <sub>2</sub> ]	[O <sub>2</sub> ]	[NO]
Initial	0.200	0.200	0.00
<b>C</b> hange	-x	-x	+2 <i>x</i>
Equil	0.200 – x	0.200 – x	2x

Step 5 Substitute the expressions for the equilibrium concentrations (from step 4) into the expression for the equilibrium constant. Using the given value of the equilibrium constant, solve the expression for the variable x. In some cases, you can take the square root of both sides of the expression to solve for x. In other cases, you must solve a quadratic equation to find x.

Remember the quadratic formula:

$$ax^{2} + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$K_{\rm c} = \frac{[\rm NO]^2}{[\rm N_2][\rm O_2]}$$
$$= \frac{(2x)^2}{(0.200 - x)(0.200 - x)}$$
$$0.10 = \frac{(2x)^2}{(0.200 - x)^2}$$
$$\sqrt{0.10} = \frac{2x}{0.200 - x}$$
$$\sqrt{0.10} (0.200 - x) = 2x$$
$$\sqrt{0.10} (0.200) - \sqrt{0.10} x = 2x$$
$$0.063 = 2x + \sqrt{0.10} x$$
$$0.063 = 2.3x$$
$$x = 0.027$$

Continued

**Step 6** Substitute x into the expressions for the equilibrium concentrations of the reactants and products (from step 4) and calculate the concentrations. In cases where you solved a quadratic and have two values for x, choose the value for x that gives a physically realistic answer. For example, reject the value of x that results in any negative concentrations.

 $[N_2] = 0.200 - 0.027$ = 0.173 M  $[O_2] = 0.200 - 0.027$ = 0.173 M [NO] = 2(0.027)= 0.054 M

Step 7 Check your answer by substituting the calculated equilibrium values into the equilibrium expression. The calculated value of K should match the given value of K. Note that rounding errors could cause a difference in the least significant digit when comparing values of the equilibrium constant.

$$K_{\rm c} = \frac{[\rm NO]^2}{[\rm N_2][\rm O_2]}$$
$$= \frac{(0.054)^2}{(0.173)(0.173)} = 0.097$$

Since the calculated value of  $K_c$  matches the given value (to within one digit in the least significant figure), the answer is valid.

Continued

#### **For Practice**

The reaction is carried out at a different temperature at which  $K_c = 0.055$ . This time, however, the reaction mixture starts with only the product, [NO] = 0.0100 M, and no reactants. Find the equilibrium concentrations of N<sub>2</sub>, O<sub>2</sub>, and NO at equilibrium.

Consider the reaction:

 $N_2O_4(g) \rightleftharpoons 2 NO_2(g)$  $K_c = 0.36 \text{ (at 100 °C)}$ 

A reaction mixture at 100 °C initially contains  $[NO_2] = 0.100$  M. Find the equilibrium concentrations of  $NO_2$  and  $N_2O_4$  at this temperature

### How To

Find Equilibrium Concentrations from Initial Concentrations and the Equilibrium Constant

To solve these types of problems, follow the given procedure.

**Step 1 Using the balanced equation as a guide, prepare a table showing the known initial concentrations of the reactants and products.** Leave room in the table for the changes in concentrations and for the equilibrium concentrations.

$$N_2O_4(g) \Longrightarrow 2 NO_2(g)$$

	[N <sub>2</sub> O <sub>4</sub> ]	[NO <sub>2</sub> ]
Initial	0.00	0.100
<b>C</b> hange		
Equil		

Continued

**Step 2** Use the initial concentrations to calculate the reaction quotient (Q) for the initial concentrations. Compare Q to K to predict the direction in which the reaction proceeds.

$$Q_{\rm c} = \frac{[\rm NO_2]^2}{[\rm N_2O_4]} = \frac{(0.100)^2}{0.00}$$
$$= \infty$$

Q > K; therefore, the reaction will proceed to the left.

**Step 3 Represent the change in the concentration of one of the reactants or products with the variable** *x***. Define the changes in the concentrations of the other reactants or products in terms of** *x***.** It is usually most convenient to let *x* represent the change in concentration of the reactant or product with the smallest stoichiometric coefficient.

$$N_2O_4(g) \Longrightarrow 2 NO_2(g)$$

	[N <sub>2</sub> O <sub>4</sub> ]	[NO <sub>2</sub> ]
Initial	0.00	0.100
Change	+x	-2x
Equil		

Continued

**Step 4** Sum each column for each reactant and each product to determine the equilibrium concentrations in terms of the initial concentrations and the variable *x*.

$$N_2O_4(g) \Longrightarrow 2 NO_2(g)$$

	[N <sub>2</sub> O <sub>4</sub> ]	[NO <sub>2</sub> ]
Initial	0.00	0.100
<b>C</b> hange	+x	-2x
Equil	х	0.100 – 2x

**Step 5 Substitute the expressions for the equilibrium concentrations (from step 4) into the expression for the equilibrium constant. Using the given value of the equilibrium constant, solve the expression for the variable x.** In some cases, you can take the square root of both sides of the expression to solve for x. In other cases, you must solve a quadratic equation to find x.

Remember the quadratic formula:

$$ax^{2} + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$K_{c} = \frac{[NO_{2}]^{2}}{[N_{2}O_{4}]}$$

$$= \frac{(0.100 - 2x)^{2}}{x}$$

$$0.36 = \frac{0.0100 - 0.400x + 4x^{2}}{x}$$

$$0.36x = 0.0100 - 0.400x + 4x^{2}$$

$$4x^{2} - 0.76x + 0.0100 = 0 (quadratic)$$

$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$= \frac{-(-0.76) \pm \sqrt{(-0.76)^{2} - 4(4)(0.0100)}}{2(4)}$$

$$= \frac{0.76 \pm 0.65}{8}$$

$$x = 0.176 \text{ or } x = 0.014$$

Continued

Step 6 Substitute x into the expressions for the equilibrium concentrations of the reactants and products (from step 4) and calculate the concentrations. In cases where you solved a quadratic and have two values for x, choose the value for x that gives a physically realistic answer. For example, reject the value of x that results in any negative concentrations.

We reject the root x = 0.176 because it gives a negative concentration for NO<sub>2</sub>. Using x = 0.014, we get the following concentrations:

$$[NO_2] = 0.100 - 2x$$
  
= 0.100 - 2(0.014) = 0.072 M  
 $[N_2O_4] = x$   
= 0.014 M

Step 7 Check your answer by substituting the calculated equilibrium values into the equilibrium expression.
 The calculated value of K should match the given value of K. Note that rounding errors could cause a difference in the least significant digit when comparing values of the equilibrium constant.

$$K_{\rm c} = \frac{[\rm NO_2]^2}{[\rm N_2O_4]} = \frac{(0.072)^2}{0.014}$$
$$= 0.37$$

Since the calculated value of  $K_c$  matches the given value (to within one digit in the least significant figure), the answer is valid.

Continued

#### **For Practice**

The reaction in Example 16.10 is carried out at the same temperature, but this time the reaction mixture initially contains only the reactant,  $[N_2O_4] = 0.0250$  M, and no NO<sub>2</sub>. Find the equilibrium concentrations of  $N_2O_4$  and NO<sub>2</sub>.

Consider the reaction:

$$I_2(g) + Cl_2(g) \Longrightarrow 2 \text{ ICl}(g) \qquad K_p = 81.9 \text{ (at 25 °C)}$$

A reaction mixture at 25 °C initially contains  $P_{I_2} = 0.100$  atm,  $P_{Cl_2} = 0.100$  atm, and  $P_{ICl} = 0.100$  atm. Find the equilibrium partial pressures of  $I_2$ ,  $Cl_2$ , and ICl at this temperature.

### **Solution**

Follow the procedure (using partial pressures in place of concentrations) to solve the problem.

Step 1 Using the balanced equation as a guide, prepare an ICE table showing the known initial partial pressures of the reactants and products.

$I_2(g) + Cl_2(g) \Longrightarrow 2 ICl(g)$			
	P <sub>l2</sub> (atm)	P <sub>Cl2</sub> (atm)	P <sub>ICI</sub> (atm)

Initial	0.100	0.100	0.100
<b>C</b> hange			
Equil			

#### Continued

**Step 2** Use the initial partial pressures to calculate the reaction quotient (Q). Compare Q to K to predict the direction in which the reaction will proceed.

$$Q_{\rm p} = \frac{(P_{\rm ICl})^2}{P_{\rm I_2}P_{\rm Cl_2}} = \frac{(0.100)^2}{(0.100)(0.100)} = 1$$
  
 $K_{\rm p} = 81.9 \text{ (given)}$   
 $Q < K$ ; therefore, the reaction will proceed to the right.

**Step 3** Represent the change in the partial pressure of one of the reactants or products with the variable x. Define the changes in the partial pressures of the other reactants or products in terms of x.

$12(3) + C1_2(3) \leftarrow 21C1(3)$				
	P <sub>l2</sub> (atm)	P <sub>Cl2</sub> (atm)	P <sub>ICI</sub> (atm)	
Initial	0.100	0.100	0.100	
Change	-x	-x	+2x	
Equil				

 $I_{\alpha}(\sigma) + CI_{\alpha}(\sigma) \Longrightarrow 2 ICI(\sigma)$ 

Continued

**Step 4** Sum each column for each reactant and product to determine the equilibrium partial pressures in terms of the initial partial pressures and the variable *x*.

 $I_2(g) + Cl_2(g) \Longrightarrow 2 ICl(g)$ 

	$P_{l_2}$ (atm)	P <sub>Cl2</sub> (atm)	P <sub>ICI</sub> (atm)
Initial	0.100	0.100	0.100
Change	—x	-x	+2 <i>x</i>
Equil	0.100 - x	0.100 - x	0.100 + 2x

Continued

**Step 5** Substitute the expressions for the equilibrium partial pressures (from step 4) into the expression for the equilibrium constant. Use the given value of the equilibrium constant to solve the expression for the variable *x*.

$$K_{\rm p} = \frac{(P_{\rm ICI})^2}{P_{\rm I_2}P_{\rm Cl_2}} = \frac{(0.100 + 2x)^2}{(0.100 - x)(0.100 - x)}$$
  

$$81.9 = \frac{(0.100 + 2x)^2}{(0.100 - x)^2} \quad \text{(perfect square)}$$
  

$$\sqrt{81.9} = \frac{(0.100 + 2x)}{(0.100 - x)}$$
  

$$\sqrt{81.9} (0.100 - x) = 0.100 + 2x$$
  

$$\sqrt{81.9} (0.100) - \sqrt{81.9} x = 0.100 + 2x$$
  

$$\sqrt{81.9} (0.100) - 0.100 = 2x + \sqrt{81.9} x$$
  

$$0.805 = 11.05x$$
  

$$x = 0.0729$$

#### Continued

**Step 6** Substitute *x* into the expressions for the equilibrium partial pressures of the reactants and products (from step 4) and calculate the partial pressures.

$$\begin{split} PI_2 &= 0.100 - 0.0729 = 0.027 \text{ atm} \\ P_{Cl_2} &= 0.100 - 0.0729 = 0.027 \text{ atm} \\ P_{ICl} &= 0.100 + 2(0.0729) = 0.246 \text{ atm} \end{split}$$

**Step 7** Check your answer by substituting the calculated equilibrium partial pressures into the equilibrium expression. The calculated value of *K* should match the given value of *K*.

 $K_{\rm p} = \frac{(P_{\rm ICl})^2}{P_{\rm I_2}P_{\rm Cl_2}} = \frac{(0.246)^2}{(0.027)(0.027)} = 83$ 

Since the calculated value of Kp matches the given value (within the uncertainty indicated by the significant figures), the answer is valid.

#### **For Practice**

The reaction between I<sub>2</sub> and Cl<sub>2</sub> is carried out at the same temperature, but with these initial partial pressures:  $P_{I_2} = 0.150$  atm,  $P_{Cl_2} = 0.150$  atm,  $P_{ICl} = 0.00$  atm. Find the equilibrium partial pressures of all three substances.

Consider the reaction for the decomposition of hydrogen disulfide:

 $2 \operatorname{H}_2 \mathcal{S}(g) \Longrightarrow 2 \operatorname{H}_2(g) + \mathcal{S}_2(g)$ 

 $K_{\rm c} = 1.67 \times 10^{-7}$  at 800 °C

A 0.500-L reaction vessel initially contains 0.0125 mol of  $H_2S$  at 800 °C. Find the equilibrium concentrations of  $H_2$  and  $S_2$ .

#### **How To**

**Determine Equilibrium Concentrations from Initial Concentrations and the Equilibrium Constant** To solve these types of problems, follow the given procedure.

Step 1 Using the balanced equation as a guide, prepare a table showing the known initial concentrations of the reactants and products. (In these examples, you first calculate the concentration of  $H_2S$  from the given number of moles and volume.)

$$[H_2S] = \frac{0.0125 \text{ mol}}{0.500 \text{ L}} = 0.0250 \text{ M}$$

$$2 \operatorname{H}_2 \mathcal{S}(g) \Longrightarrow 2 \operatorname{H}_2(g) + \operatorname{S}_2(g)$$

	[H <sub>2</sub> S]	[H <sub>2</sub> ]	[ <b>S</b> <sub>2</sub> ]
Initial	0.0250	0.00	0.00
Change			
Equil			

#### Continued

**Step 2** Use the initial concentrations to calculate the reaction quotient (Q). Compare Q to K to predict the direction in which the reaction proceeds.

By inspection, Q = 0; the reaction proceeds to the right.

**Step 3** Represent the change in the concentration of one of the reactants or products with the variable *x*. Define the changes in the concentrations of the other reactants or products with respect to *x*.

2 (0)		2(0)	2(0)
	[H <sub>2</sub> S]	[H <sub>2</sub> ]	[S <sub>2</sub> ]
Initial	0.0250	0.00	0.00
<b>C</b> hange	-2x	+2x	+x
Equil			

 $2 \operatorname{H}_2 \mathcal{S}(g) \Longrightarrow 2 \operatorname{H}_2(g) + \mathcal{S}_2(g)$ 

#### Continued

**Step 4** Sum each column for each reactant and product to determine the equilibrium concentrations in terms of the initial concentrations and the variable *x*.

			2(0)
	[H <sub>2</sub> S]	[H <sub>2</sub> ]	[ <b>S</b> <sub>2</sub> ]
Initial	0.0250	0.00	0.00
<mark>C</mark> hange	-2x	+2x	+x
Equil	0.0250 - 2x	2x	х

 $2 \operatorname{H}_2 \mathcal{S}(g) \Longrightarrow 2 \operatorname{H}_2(g) + \mathcal{S}_2(g)$ 

**Step 5** Substitute the expressions for the equilibrium concentrations (from step 4) into the expression for the equilibrium constant. Use the given value of the equilibrium constant to solve the resulting equation for the variable x. In this case, the resulting equation is cubic in x. Although cubic equations can be solved, the solutions are not usually simple. However, since the equilibrium constant is small, we know that the reaction does not proceed very far to the right. Therefore, x is a small number and can be dropped from any quantities in which it is added to or subtracted from another number (as long as the number itself is not too small).

Continued

Check whether your approximation is valid by comparing the calculated value of x to the number it was added to or subtracted from. The ratio of the two numbers should be less than 0.05 (or 5%) for the approximation to be valid. If the approximation is not valid, proceed to step 5a.

$$K_{c} = \frac{[H_{2}]^{2}[S_{2}]}{[H_{2}S]^{2}}$$

$$= \frac{(2x)^{2}x}{(0.0250 - 2x)^{2}}$$

$$1.67 \times 10^{-7} = \frac{4x^{3}}{(0.0250 - 2x)^{2}}$$

$$1.67 \times 10^{-7} = \frac{4x^{3}}{(0.0250 - 2x)^{2}}$$

$$1.67 \times 10^{-7} = \frac{4x^{3}}{6.25 \times 10^{-4}}$$

$$6.25 \times 10^{-4}(1.67 \times 10^{-7}) = 4x^{3}$$

$$x^{3} = \frac{6.25 \times 10^{-4}(1.67 \times 10^{-7})}{4}$$

$$x = 2.97 \times 10^{-4}$$

Continued

Checking the *x* is small approximation:

 $\frac{2.97 \times 10^{-4}}{0.0250} \times 100\% = 1.19\%$ 

The *x* is small approximation is valid; proceed to step 6.

Step 5a If the approximation is not valid, you can either solve the equation exactly (by hand or with your calculator), or use the method of successive approximations. In this example, the method of successive approximations is used.

Substitute the value obtained for x in step 5 back into the original cubic equation, but only at the exact spot where you assumed x was negligible, and then solve the equation for x again. Continue this procedure until the value of x you obtain from solving the equation is the same as the one that you substituted into the equation.

#### Continued

**Step 6** Substitute *x* into the expressions for the equilibrium concentrations of the reactants and products (from step 4) and calculate the concentrations.

 $[H_2S] = 0.0250 - 2(2.97 \times 10^{-4})$ = 0.0244 M $[H_2] = 2(2.97 \times 10^{-4})$  $= 5.94 \times 10^{-4} M$  $[S_2] = 2.97 \times 10^{-4} M$ 

**Step 7** Check your answer by substituting the calculated equilibrium values into the equilibrium expression. The calculated value of *K* should match the given value of *K*. Note that the approximation method and rounding errors could cause a difference of up to about 5% when comparing values of the equilibrium constant.

$$K_{\rm c} = \frac{(5.94 \times 10^{-4})^2 (2.97 \times 10^{-4})}{(0.0244)^2}$$
$$= 1.76 \times 10^{-7}$$

The calculated value of K is close enough to the given value when you consider the uncertainty introduced by the approximation. Therefore, the answer is valid.

Continued

### **For Practice**

The reaction is carried out at the same temperature with the following initial concentrations:  $[H_2S] = 0.100 \text{ M}, [H_2] = 0.100 \text{ M}, \text{ and } [S_2] = 0.00 \text{ M}.$  Find the equilibrium concentration of  $[S_2]$ .

Consider the reaction for the decomposition of hydrogen disulfide:

$$2 \operatorname{H}_2 \mathcal{S}(g) \Longrightarrow 2 \operatorname{H}_2(g) + \mathcal{S}_2(g)$$

 $K_{\rm c} = 1.67 \times 10^{-7}$  at 800 °C

A 0.500-L reaction vessel initially contains 1.25 × 10<sup>-4</sup> mol of H<sub>2</sub>S at 800 °C. Find the equilibrium concentrations of H<sub>2</sub> and S<sub>2</sub>.

### **How To**

**Determine Equilibrium Concentrations from Initial Concentrations and the Equilibrium Constant** To solve these types of problems, follow the given procedure.

Step 1 Using the balanced equation as a guide, prepare a table showing the known initial concentrations of the reactants and products. (In these examples, you first calculate the concentration of  $H_2S$  from the given number of moles and volume.)  $1.25 \times 10^{-4}$  mol

$$H_2S] = \frac{1.25 \times 10^{-4} \text{ mol}}{0.500 \text{ L}}$$
$$= 2.50 \times 10^{-4} \text{ M}$$

$$2 \operatorname{H}_2 S(g) \Longrightarrow 2 \operatorname{H}_2(g) + S_2(g)$$

	[H <sub>2</sub> S]	[H <sub>2</sub> ]	[ <b>S</b> <sub>2</sub> ]
Initial	$2.50 \times 10^{-4}$	0.00	0.00
Change			
Equil			

#### Continued

**Step 2** Use the initial concentrations to calculate the reaction quotient (Q). Compare Q to K to predict the direction in which the reaction proceeds.

By inspection, Q = 0; the reaction proceeds to the right.

**Step 3** Represent the change in the concentration of one of the reactants or products with the variable x. Define the changes in the concentrations of the other reactants or products with respect to x.

2 (0)				
	[H <sub>2</sub> S]	[H <sub>2</sub> ]	[S <sub>2</sub> ]	
Initial	$2.50 imes10^{-4}$	0.00	0.00	
<b>C</b> hange	-2x	+2x	+ <b>x</b>	
Equil				

 $2 \operatorname{H}_2 \mathrm{S}(g) \Longrightarrow 2 \operatorname{H}_2(g) + \operatorname{S}_2(g)$ 

#### Continued

**Step 4** Sum each column for each reactant and product to determine the equilibrium concentrations in terms of the initial concentrations and the variable *x*.

	[H <sub>2</sub> S]	[H <sub>2</sub> ]	[S <sub>2</sub> ]	
Initial	$2.50 \times 10^{-4}$	0.00	0.00	
<b>C</b> hange	-2x	+2x	+x	
Equil	$2.50 \times 10^{-4} - 2x$	2x	x	

 $2 \operatorname{H}_2 S(g) \Longrightarrow 2 \operatorname{H}_2(g) + S_2(g)$ 

**Step 5** Substitute the expressions for the equilibrium concentrations (from step 4) into the expression for the equilibrium constant. Use the given value of the equilibrium constant to solve the resulting equation for the variable x. In this case, the resulting equation is cubic in x. Although cubic equations can be solved, the solutions are not usually simple. However, since the equilibrium constant is small, we know that the reaction does not proceed very far to the right. Therefore, x is a small number and can be dropped from any quantities in which it is added to or subtracted from another number (as long as the number itself is not too small).

Continued

Check whether your approximation is valid by comparing the calculated value of x to the number it was added to or subtracted from. The ratio of the two numbers should be less than 0.05 (or 5%) for the approximation to be valid. If the approximation is not valid, proceed to step 5a.

$$K_{c} = \frac{[H_{2}]^{2}[S_{2}]}{[H_{2}S]^{2}}$$

$$= \frac{(2x)^{2}x}{(2.50 \times 10^{-4} - 2x)^{2}}$$

$$1.67 \times 10^{-7} = \frac{4x^{3}}{(2.50 \times 10^{-4} - 2x)^{2}}$$

$$1.67 \times 10^{-7} = \frac{4x^{3}}{(2.50 \times 10^{-4} - 2x)^{2}}$$

$$1.67 \times 10^{-7} = \frac{4x^{3}}{(2.50 \times 10^{-4} - 2x)^{2}}$$

$$1.67 \times 10^{-7} = \frac{4x^{3}}{6.25 \times 10^{-8}}$$

$$6.25 \times 10^{-8}(1.67 \times 10^{-7}) = 4x^{3}$$

$$x^{3} = \frac{6.25 \times 10^{-8}(1.67 \times 10^{-7})}{4}$$

$$x = 1.38 \times 10^{-5}$$

Continued

Checking the *x* is *small* approximation:

 $\frac{1.38 \times 10^{-5}}{2.50 \times 10^{-4}} \times 100\% = 5.52\%$ 

The approximation does not satisfy the <5% rule (although it is close).

Step 5a If the approximation is not valid, you can either solve the equation exactly (by hand or with your calculator), or use the method of successive approximations. In this example, the method of successive approximations is used.

Continued

Substitute the value obtained for x in step 5 back into the original cubic equation, but only at the exact spot where you assumed x was negligible, and then solve the equation for x again. Continue this procedure until the value of x you obtain from solving the equation is the same as the one that you substituted into the equation.

$$1.67 \times 10^{-7} = \frac{4x^3}{(2.50 \times 10^{-4} - 2x)^2}$$

$$x = 1.38 \times 10^{-5}$$

$$1.67 \times 10^{-7} = \frac{4x^3}{(2.50 \times 10^{-4} - 2.76 \times 10^{-5})^2}$$
$$x = 1.27 \times 10^{-5}$$

If we substitute this value of x back into the cubic equation and solve it, we get  $x = 1.28 \times 10^{-5}$ , which is nearly identical to  $1.27 \times 10^{-5}$ . Therefore, we have arrived at the best approximation for x.

#### Continued

**Step 6** Substitute *x* into the expressions for the equilibrium concentrations of the reactants and products (from step 4) and calculate the concentrations.

 $[H_2S] = 2.50 \times 10^{-4} - 2(1.28 \times 10^{-5})$  $= 2.24 \times 10^{-4} M$  $[H_2] = 2(1.28 \times 10^{-5})$  $= 2.56 \times 10^{-5} M$  $[S_2] = 1.28 \times 10^{-5} M$ 

**Step 7** Check your answer by substituting the calculated equilibrium values into the equilibrium expression. The calculated value of K should match the given value of K. Note that the approximation method and rounding errors could cause a difference of up to about 5% when comparing values of the equilibrium constant.

$$K_{\rm c} = \frac{(2.56 \times 10^{-5})^2 (1.28 \times 10^{-5})}{(2.24 \times 10^{-4})^2}$$
$$= 1.67 \times 10^{-7}$$

The calculated value of K is equal to the given value. Therefore, the answer is valid.

Continued

#### **For Practice**

The reaction is carried out at the same temperature with the following initial concentrations:  $[H_2S] = 1.00 \times 10^{-4} M$ ,  $[H_2] = 0.00 M$ , and  $[S_2] = 0.00 M$ . Find the equilibrium concentration of  $[S_2]$ .

### The Effect of a Concentration Change on Equilibrium

Consider the following reaction at equilibrium:

 $CaCO_3(s) \Longrightarrow CaO(s) + CO_2(g)$ 

What is the effect of adding additional CO<sub>2</sub> to the reaction mixture? What is the effect of adding additional CaCO<sub>3</sub>?

#### **Solution**

Adding additional  $CO_2$  increases the concentration of  $CO_2$  and causes the reaction to shift to the left. Adding additional  $CaCO_3$ , however, does *not* increase the concentration of  $CaCO_3$  because  $CaCO_3$  is a solid and therefore has a constant concentration. Thus, adding additional  $CaCO_3$  has no effect on the position of the equilibrium. (Note that, as we saw in Section 16.5, solids are not included in the equilibrium expression.)

#### **For Practice**

Consider the following reaction in chemical equilibrium:

 $2 \operatorname{BrNO}(g) \Longrightarrow 2 \operatorname{NO}(g) + \operatorname{Br}_2(g)$ 

What is the effect of adding additional Br<sub>2</sub> to the reaction mixture? What is the effect of adding additional BrNO?

### The Effect of a Volume Change on Equilibrium

Consider the following reaction at chemical equilibrium:

 $2 \operatorname{KClO}_3(s) \rightleftharpoons 2 \operatorname{KCl}(s) + 3 \operatorname{O}_2(g)$ 

What is the effect of decreasing the volume of the reaction mixture? Increasing the volume of the reaction mixture? Adding an inert gas at constant volume?

#### **Solution**

The chemical equation has 3 mol of gas on the right and zero moles of gas on the left. Decreasing the volume of the reaction mixture increases the pressure and causes the reaction to shift to the left (toward the side with fewer moles of gas particles). Increasing the volume of the reaction mixture decreases the pressure and causes the reaction to shift to the right (toward the side with more moles of gas particles.) Adding an inert gas has no effect.

### **For Practice**

Consider the following reaction at chemical equilibrium:

$$2 \operatorname{SO}_2(g) + \operatorname{O}_2(g) \Longrightarrow 2 \operatorname{SO}_3(g)$$

What is the effect of decreasing the volume of the reaction mixture? Increasing the volume of the reaction mixture?

### The Effect of a Temperature Change on Equilibrium

The following reaction is endothermic:

 $CaCO_3(s) \Longrightarrow CaO(s) + CO_2(g)$ 

What is the effect of increasing the temperature of the reaction mixture? Decreasing the temperature?

#### **Solution**

Since the reaction is endothermic, we can think of heat as a reactant:

Heat + 
$$CaCO_3(s) \implies CaO(s) + CO_2(g)$$

Raising the temperature is like adding a reactant, causing the reaction to shift to the right. Lowering the temperature is equivalent to removing a reactant, causing the reaction to shift to the left.

#### **For Practice**

The following reaction is exothermic:

$$2 \operatorname{SO}_2(g) + \operatorname{O}_2(g) \rightleftharpoons 2 \operatorname{SO}_3(g)$$

What is the effect of increasing the temperature of the reaction mixture? Decreasing the temperature?